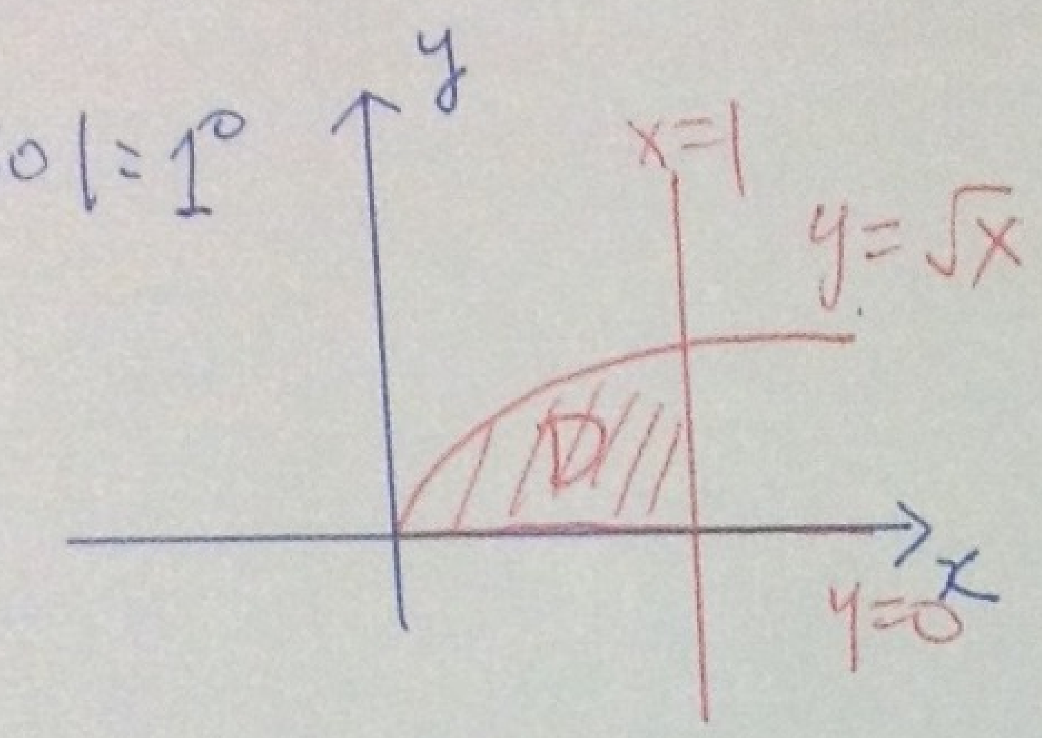
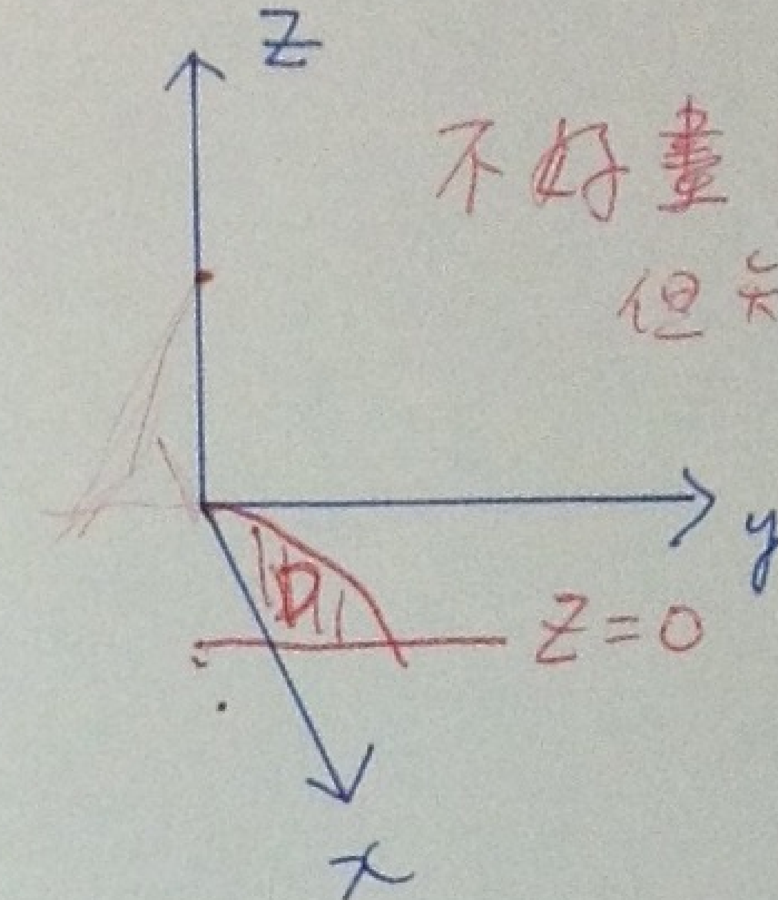
§4.5 $\iiint_E f\,dV$

#9. $\iiint_E 6xy\,dV$, where E lies under the plane $z=1+x+y$ and above the region in xy-plane ($z=0$) bdd by the curves $y=\sqrt{x}$, $y=0$ and $x=1$.

sol: 1°





不好畫 $z=1+x+y$
但知其在D上 $z>0$

故 $E=\begin{cases} 0\le z\le 1+x+y \\ (x,y)\in D \end{cases}$

$D=\begin{cases} 0\le x\le 1 \\ 0\le y\le \sqrt{x} \end{cases}$

2° $\iiint_E 6xy\,dV = \int_0^1\int_0^{\sqrt{x}}\left(\int_0^{1+x+y} 6xy\,dz\right)dy\,dx$

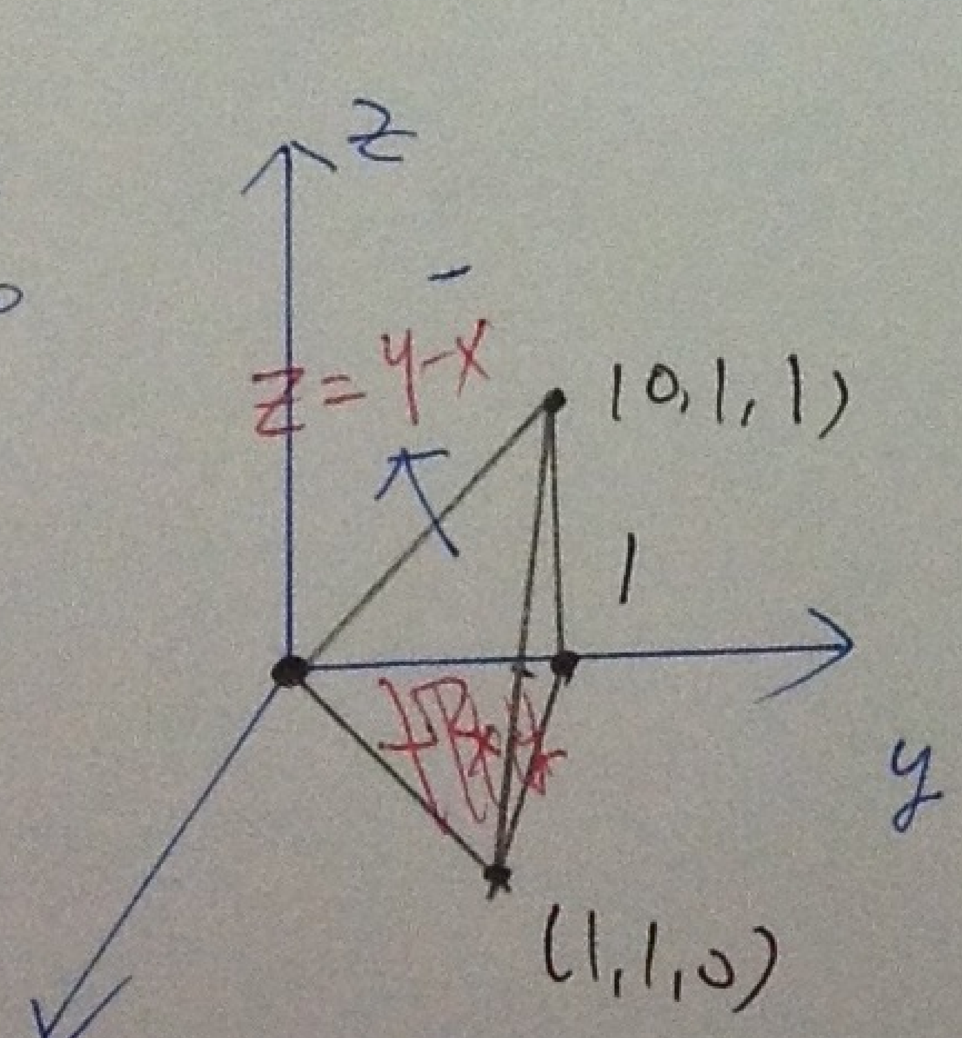
$= \int_0^1 \left(3xy^2+3x^2y^2+2xy^3\Big|_{y=0}^{y=\sqrt{x}}\right)dx$

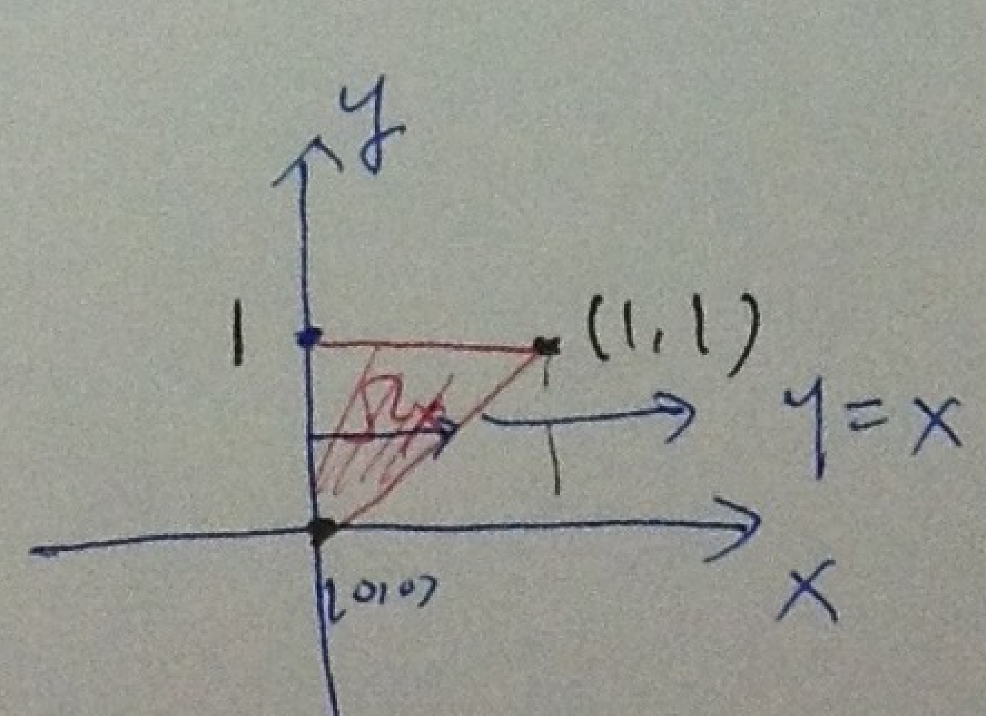
$= \int_0^1 (3x^2+3x^3+2x^{\frac{5}{2}})\,dx = \underline{\frac{65}{28}}$ #

#12. $\iiint_E xz\,dV$, E is tetrahedron with vertices $(0,0,0)$, $(0,1,0)$, $(1,1,0)$ and $(0,1,1)$

sol:

1°





$E:\begin{cases} (x,y)\in\Omega_{xy} \\ 0\le z\le y-x \end{cases}$ $\quad\Omega_{xy}:\begin{cases} 0\le y\le 1 \\ 0\le x\le y \end{cases}$

過 $(0,0,0)$ $(1,1,0)$ $(0,1,1)$

三点之平面 eq 為 $x-y+z=0$

2° $\iiint_E xz\,dV = \int_0^1\left[\int_0^y\left(\int_0^{y-x} xz\,dz\right)dx\right]dy = \underline{\frac{1}{120}}$ #